



## A Review of H-B-T Correlations and What We Can Learn About The Space-Time Structure of Hadronic Collisions. <sup>1</sup>

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### ABSTRACT

In this talk I review Hanbury-Brown-Twiss particle interferometry as a tool which is useful for understanding the space-time dynamics of ultra-relativistic nuclear collisions.

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## 1 Introduction

In this talk, I shall give a pedagogical review of general features of Hanbury-Brown-Twiss pion interferometry.<sup>1-5</sup> This experimental technique involves measuring correlations in the two particle emission of pions. The degree of correlation is determined by comparing to single particle inclusive distribution functions. The technique may be used not only for pions but for any two identical species of particles.

The two particle correlation arises from the interference of the particle wavefunctions, and depends on whether the particles are bosons or fermions. As we shall see, the degree of interference depends upon the degree of coherence of the emitting source of the particles, and is maximal for a totally incoherent source. The maximization of the interference from a totally incoherent source is counter intuitive, and I shall try to explain this in detail in the talk.

Perhaps the primary reason that Hanbury-Brown-Twiss pion interferometry is so interesting is that it allows for a determination of the size and time scales of the region where the particles stop talking to one another. Technically, this region is called the decoupling volume. What it means in practice is that in the nucleus-nucleus collision, there is a matter distribution produced, and the constituents scatter from one another for some time. Perhaps there is enough scattering to produce a thermalized distribution function, or perhaps not. For each particle, there is some more or less well defined space-time point which corresponds to its last scattering. The collection of these points, defines the decoupling space-time volume.

In Fig. 1, a space-time picture of ultra-relativistic nuclear collisions is shown.

The initial nuclei are incident upon the light cone axis  $t = z$  and  $t = -z$ . The nuclei collide at  $t = z = 0$ . This collision point is well defined up to the quantum mechanical uncertainty in the longitudinal positions of the wee partons of the nuclei. At very high energies, this should correspond to a distance scale of  $d \leq 1 \text{ Fm}$ . At CERN energies, the nuclei are much less Lorentz contracted since the energy is not so high.

In the standard picture of high energy nuclear collisions, the matter is assumed to thermalize at some proper time,  $\tau = \sqrt{t^2 - z^2}$ ,  $\tau = \tau_{\text{formation}}$ . The matter is then assumed to hydrodynamically expand until a proper time  $\tau_{\text{decoupling}}$ , and there is therefore a well defined decoupling surface.

The standard picture of ultra-relativistic nuclear collisions can be used to simply predict pion H-B-T distribution functions, and has been computed by Makhlin-Sinyukov and Kolehmainen-Gyulassy<sup>6-7</sup>. We shall refer to this as the GMKS (gimmicks) model. Its advantage is that it analytically parameterizes the correlation function, and many generic features may be directly studied. Its disadvantage is that it is somewhat unrealistic. The particles emitted at decoupling are not emitted at a well defined time. As we shall see, there are large fluctuations. The surface becomes therefore very spread out, and this spread is reflected in the correlations. Also, any effects of the transverse evolution are ignored in the GMKS model.

The simplest example of a H-B-T interference is shown in Fig. 2. Here there are two sources at  $r_1$  and  $r_2$ . The identical particles are detected at position  $x$ . There are two possible paths the particles may take from  $r_1, r_2$  to  $x$ . The paths are different because the momenta of the particle are different. These paths are shown by the solid lines and by the dashed lines.

To measure the degree of correlation, let the single particle distribution function be

$$\frac{dN}{d^3k} \quad (1)$$

The two particle distribution function is

$$\frac{dN}{d^3k d^3k'} \quad (2)$$

The correlation is measured by

$$R(k, k') = \frac{dN}{d^3k d^3k'} / \frac{dN}{d^3k} \frac{dN}{d^3k'} \quad (3)$$

When there is no correlation, this ratio goes to 1.

To compute the correlation, we will write the emission amplitude as

$$A(k) = \frac{1}{\sqrt{2}} \rho(k) (e^{i\phi} e^{ikx} \pm e^{i\phi'} e^{ikx'}) \quad (4)$$

In this equation  $\rho(k)$  is a real function which characterizes the source strength, which we take to be equal for the two sources. The phases  $\phi$  are taken to be different between the two sources, and we also ignore any  $k$  dependence of the phases. The  $\pm$  sign ambiguity is  $+$  for bosons and  $-$  for fermions. The single particle distribution is

$$\frac{dN}{d^3k} = \rho(k)^2 (1 \pm \cos[k \cdot (x - x') + \phi + \phi']) \quad (5)$$

When there is incoherent emission, we have

$$\langle e^{i\phi - i\phi'} \rangle = 0 \quad (6)$$

so that

$$\frac{dN}{d^3k} = \rho(k)^2 \quad (7)$$

The amplitude for the two particle emission is given by

$$A(k, k') = \frac{1}{\sqrt{2}} \rho(k) \rho(k') [e^{ikx + ik'x' + i\phi + i\phi'} \pm e^{ik'x + ikx' + i\phi + i\phi'}] \quad (8)$$

The two particle distribution function is

$$\frac{dN}{d^3k d^3k'} = \rho(k)^2 \rho(k')^2 (1 \pm \cos[(k - k')(x - x')]) \quad (9)$$

which is independent of  $\phi$  and  $\phi'$ .

The correlation function  $R$  for this example is therefore

$$R(k, k') = 1 \pm \cos[(k - k')(x - x')] \quad (10)$$

The correlation function has a characteristic oscillation scale of order  $\Delta k \sim 1/\Delta x$ . Notice that  $x - x'$  is just the distance of separation of the sources. The correlation function (In this example the correlation function does not approach one at large relative momenta, and oscillates. This is an artifact of having only two sources, and for realistic source distributions the correlation function goes to one. The rapidly oscillating cosine may be thought of as zero for large relative momenta in the sense

of a distribution function.) At zero relative momenta, the correlation function goes to  $1 \pm 1$ . A realistic distribution is shown in Fig.3 for the two cases of boson and fermion correlation.

Notice in this example, the correlations arose for incoherent particle emission. This is a general feature of H-B-T correlations. We shall soon see that for totally coherent emission, the correlation function is identically one, that is there is no correlation!

## 2 Correction to the Correlation Function from Final State Interactions

The assumption that after emission from the source, the pions do not interact is of course an approximation. If the separation between the sources is large, then the short ranged nuclear force is not so important. In realistic nuclear collisions, of course these corrections are non-negligable and must be computed. In computations which as of the date of this meeting which have been compared to experiment, such corrections have not been included.

A correction due to final state interactions which is non-negligable whatever the source separation is the long ranged Coulomb interaction. This correction is important for the small  $\Delta k$  contribution to the correlation function. It has been already taken into account in the data presented at this meeting from the NA35 collaboration.<sup>8</sup>

The magnitude of the Coulomb correction is scaled by the dimensionless variable

$$2\pi\eta = \frac{2\pi\alpha m}{\Delta k} \quad (11)$$

For pions,  $2\pi\alpha m_\pi \sim 7\text{Mev}$  This corresponds to a size scale of about 30 Fm, and is in fact a substantial correction for the NA35 data. For kaons or protons, the correction due to Coulomb final state interactions is obviously more substantial.

In the classic analysis of Gyulassy et. al.,<sup>9</sup> it was shown that final state Coulomb correction modify the correlation function  $R$  as

$$R(k, k') = \frac{2\pi\eta}{e^{2\pi\eta} - 1} R_0 \quad (12)$$

The effect of Coulomb correcting the correlation function is shown in Fig. 4. The correction causes the experimentally observed correlation function to go to zero at  $\Delta k = 0$ .

### 3 The Correlation Function for Large Number of Emitted Mesons

In this section, I will discuss the structure of the correlation function when a large number of mesons are emitted. I will consider  $N$  emission points where the coordinate of each emission point is  $x_i$ . I again ignore a possible momentum dependence for the phase of the emission amplitude, although it would be straightforward to include such a dependence if necessary.

The amplitude for emission of a meson is

$$A(k) = \sum_{i=1}^N e^{i\phi_i} e^{ikx_i} \rho_i(k) \quad (13)$$

I define

$$P_1(k) = \frac{dN}{d^3k} \quad (14)$$

and

$$P_2(k, k') = \frac{dN}{d^3k d^3k'} \quad (15)$$

Using these definitions, we find

$$P_1(k) = \sum_{i,j} \rho_i(k) \rho_j(k) e^{ik(x_i - x_j)} \langle e^{i\phi_i - i\phi_j} \rangle \quad (16)$$

and

$$P_2(k, k') = \sum_{i,j,k,l} \langle e^{i\phi_i + i\phi_j - i\phi_k - i\phi_l} \rangle e^{ikx_i + ik'x_j - ikx_k - ik'x_l} \rho_i(k) \rho_j(k') \rho_k(k) \rho_l(k') \quad (17)$$

There are two cases where this formula simplifies. The first is the case of total phase coherence, where all of the phases are the same. The second is total phase incoherence, where all of the phases at the various emission points are random.

Consider first the case where all of the phases are coherent,  $\phi_i = \phi$ . Defining

$$\Delta(k) = \sum_i e^{ikx_i} \rho_i(k) \quad (18)$$

we have

$$P_1(k) = |\Delta(k)|^2 \quad (19)$$

and

$$P_2(k, k') = |\Delta(k)|^2 |\Delta(k')|^2 \quad (20)$$

We see that in this limit,

$$R(k, k') = 1 \quad (21)$$

Therefore, for totally coherent emission, the pions are totally uncorrelated.

The other simple case is when the pions are totally incoherent. In this case we have

$$\langle e^{i\phi_i - i\phi_j} \rangle = \delta_{ij} \quad (22)$$

and

$$\langle e^{i\phi_i + \phi_j - \phi_k - \phi_l} \rangle = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \quad (23)$$

Using these relations in the definitions of  $P_1$  and  $P_2$ , we find

$$P_1(k) = \sum_i |\rho_i(k)|^2 \quad (24)$$

and

$$P_2(k, k') = P_1(k)P_1(k') + |G(k, k')|^2 \quad (25)$$

where

$$G(k, k') = \sum_i e^{i(k-k')x_i} \rho_i(k) \rho_i(k') \quad (26)$$

Notice that from the definition of  $G$  that

$$G(k, k) = P_1(k) \quad (27)$$

and

$$\lim_{\Delta k \rightarrow \infty} G(k, k') \rightarrow 0 \quad (28)$$

The correlation function

$$R(k, k') = 1 + \frac{|G(k, k')|^2}{P_1(k)P_1(k')} \quad (29)$$

The general structure of this correlation has been discussed before, and is shown in Fig. 3. The correlation function goes to two at  $\Delta k = 0$ , and is 1 for  $\Delta k \geq 1/R$  where  $R$  is the size of the emission volume.

What are the effects of coherence for the emission of a large number of pions? In an attempt to provide a qualitative answer to that question, we shall assume that the correlations in phase are spatially local. I will first show that if there are only irreducible two phase correlations, then the correlation function goes to 2 at  $\Delta k = 0$ . For only irreducible two phase correlations,

$$\langle e^{i(\phi_i - \phi_j)} \rangle = C_{ij} \quad (30)$$

The correlation function  $C_{ij}$  is assumed to fall off rapidly when  $x_i$  and  $x_j$  are far separated in space and time. We assume there is some coherence length associated with this fall off,  $L_{coh}$ . The reducible four phase correlation function may be written in terms of the irreducible four phase correlation function as

$$\langle e^{i(\phi_i + \phi_j - \phi_k - \phi_l)} \rangle = C_{ik}C_{jl} + C_{il}C_{jk} \quad (31)$$

When these relations are used in the definitions of  $P_1$  and  $P_2$ , it is straightforward to show that  $R = 2$  when  $\Delta k = 0$ .

The irreducible two phase correlation do however affect the shape of the distribution given by  $R(k, k')$ . The detailed functional form is modified, and cannot be simply extracted. We should note that these correlation effects are most significant for pions. For kaons and nucleons, the number of produced particles, at least in the central region of ultra-relativistic nuclear collisions, is not so large. Therefore we expect that these more massive particles will be separated at emission by a distance larger than  $L_{coh}$ .

When irreducible 4 phase correlations are taken into account, the magnitude of the correlation function is reduced from 2 at  $\Delta k = 0$ . Recall that such a correlation function  $C_{ijkl}$  must vanish when any of the  $x$ 's are far separated from the others. It is therefore easy to estimate the magnitude of such a correction to  $P_1$  and to  $P_2$ . For  $P_1$ , we have

$$\delta P_2 \sim \left( \frac{dN}{dy} \right)^2 \left( \frac{V_{coh}}{V} \right)^2 \sim \left( \frac{V_{coh}}{1Fm} \right)^2 \quad (32)$$

This correction to  $P_1$  is already included in the definition of the irreducible two phase correlation function correction. All that we have done here is to show that the correction is non-vanishing as  $V_{coh}/V \rightarrow 0$ . The function  $P_4$  gets a similar contribution from the irreducible two phase correlation function, and these corrections to  $P_1$  and  $P_2$  combine together to yield no suppression of  $R$  at  $\Delta k = 0$ .

The non-trivial contribution to  $P_4$  arising from the irreducible four phase correlation is

$$\delta P_4 \sim - \left( \frac{dN}{dy} \right)^2 \left( \frac{V_{coh}}{V} \right)^3 \sim - \left( \frac{V_{coh}}{1Fm} \right)^2 \frac{V_{coh}}{V} \quad (33)$$

Notice that this correction is negative, and can be seen to reduce the value of  $R(k, k) \leq 2$ .

This correction however vanishes as  $V_{coh}/V \rightarrow 0$ . For a pp collision, such a correction should be of order one, since the emission volume is of the order of size of the correlation length. In a nucleus-nucleus collision, in the limit of large nuclear size,  $R(k, k) = 2$ . This must be true! Because nuclear collisions involve much larger collision volumes, the effects of particle coherence must be much smaller than is the case for pp collisions.

## 4 The GMKS (Gimmicks) Result for the Inside-Outside Cascade

The inside-outside cascade model for hadron production assumes a distribution of pions which is invariant under boosts along the beam axis. Measuring times and distances from the collision of the nuclei at  $t = z = 0$ , the decoupling is assumed to occur at a Lorentz invariant proper time  $\tau_{dec}$ , where  $\tau = \sqrt{t^2 - z^2}$ . The space-time rapidity,

$$\eta = \frac{1}{2} \ln \left( \frac{t+z}{t-z} \right) \quad (34)$$

is assumed to equal the momentum space rapidity,

$$y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right) \quad (35)$$

that is  $y = \eta$ , for the final state pions. At the decoupling time, the pions are assumed to be emitted in a cylinder of radius the nuclear radius at a temperature determined by the decoupling time and the multiplicity. We shall take the decoupling temperature and the final time to be independent parameters in what follows, although one can be computed in terms of the other using Bjorken's hydrodynamic model.

We now have all the information to construct the correlation function  $R$ . The space-time volume is assumed to be defined by the decoupling time, a uniform distribution in space-time rapidity, and a gaussian distribution with an extent of the

nuclear radius. The momentum space distributions are assumed to be uniform in rapidity, and thermal in transverse momenta. (In order to get an analytic parameterization, the distributions have to be modified a slight bit from thermal, but this is inconsequential for the physics of the result).

The result of such a computation is

$$R(k, k') = 1 + e^{-q_t^2 R^2/2} \frac{|K_0(\sqrt{u})|^2}{K_0(M_t/T)K_0(M'_t/T)} \quad (36)$$

where the variable  $u$  is

$$u = 2M_t M'_t (\tau_{dec}^2 + 1/4T^2) \cosh(\Delta y) + (M_t^2 + M'^2_t)(1/4T^2 - \tau_{dec}^2) + i\tau_{dec}(M_t^2 - M'^2_t)/T \quad (37)$$

There are several interesting qualitative features of this distribution function. For very heavy particles, it becomes

$$R \sim 1 + e^{-q_t^2 R^2/2} e^{-MT\tau_{dec}^2 \Delta y^2/2} \quad (38)$$

The rapidity correlation length is therefore of order  $1/\tau_{dec}\sqrt{MT} \sim z_{dec}/\tau_{dec}$ . For the case of small mass pions,  $z_{dec} \sim 1/T$ . If coherence effects are important, as we expect for pions, then the factor of  $1/T$  becomes replaced by a typical particle coherence length.

The  $q_t$  dependence is somewhat amusing. We consider the case of pions. There are two separate possibilities. In the first, we take  $q_t$  to be orthogonal to  $(p_1 + p_2)_t$ . In this case, the scale of fall off in  $q_t$  is controlled by  $1/R$ . In the other case, where  $q_t$  is parallel to  $(p_1 + p_2)_t$ , then the falloff is controlled by  $1/\sqrt{R^2 + \tau_{dec}^2}$ . This point has been emphasized by Hama.<sup>10-11</sup> To get a clear determination of the transverse radius, and the decoupling time separately, these two components must be isolated.

## 5 H-B-T Pion Correlations in a Cascade Computation

There has been recently an attempt to compute the H-B-T correlation function when the final state interaction of pions produced in the nuclear collision are more properly taken into account.<sup>12</sup> In this cascade model, the transition from hadron gas to plasma is modeled by taking a distribution of plasma globs, and letting them decay into pions. The globs can emit and reabsorb pions as black bodies. In a

non-expanding system, this would guarantee that the plasma globs and the pions would come into thermal equilibrium at the temperature of emission of pions from the globs. In an expanding system, the distribution is more inhomogeneous, as it should be. The emitted pions are allowed to scatter from one another with cross sections determined from experiment.

Such a computation has many advantages over the results gotten in the GMKS model. The decoupling time is not fixed, and there is a distribution of times. The correlation between space-time rapidity and momentum space-rapidity is similarly spread out. The transverse coordinates of emission also have a distribution. These distributions are shown in Figs. 5a -5c.<sup>12</sup>

## 6 Miscellaneous Phenomenon Which Can Make $R(k, k) \leq 2$

As we mentioned above, one mechanism for having  $R(k, k) \leq 2$  is by having coherent emission of pions. To have this be a significant effect, the emission must be coherent over the entire nuclear volume. Needless to say, if this were true, we would have to radically re-assess our current understanding of ultra-relativistic nuclear collisions.<sup>13</sup>

Another way to reduce the correlation function is by having a significant contamination from mis-identified particles. Mis-identified particles of different species would not lead to a correlation at small  $\Delta k$ , and would therefore reduce the intercept of the correlation function.

Another possible effect is uncomputed final state interactions, other than Coulombic. Such final state interactions can in principle shift the correlation function up or down. For a large source, such interactions should be of decreasing importance.

Yet another possibility is narrow resonance decay. Here a narrow resonance decays at late times far from the collision region. One of its decay products interferes with a directly produced particle. Therefore,  $\Delta k \sim 1/\tau$  where  $\tau$  is the particle lifetime. This leads to a narrow spike for small  $k$ . For the CERN experiments, such resonances would have to have a width of less than 40 Mev, to contaminate the result, and they would have to be copiously produced. We would naively expect that narrow resonances would be most copiously produced in the central region. It

is unlikely they could shift the intercept by a large amount, but their effect must be estimated properly before concluding there is a serious discrepancy with the data.

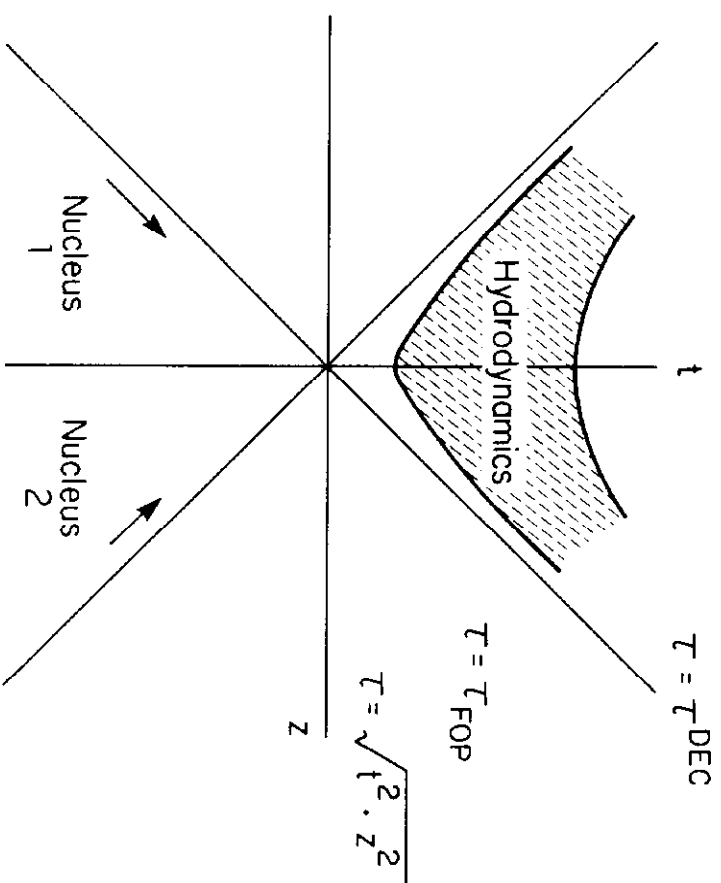
Finally, fits to experimental data are often most heavily weighted where error bars are the smallest. This is typically far away from  $\Delta k = 0$ . The fit is typically made to a theoretical distribution function. In the GMKS model, the particles are assumed to be emitted from a well defined decoupling surface. Cascade simulations indicate that this assumption is a very crude one. The shape of the correlation function may in fact not be generally well fit by a Gaussian.

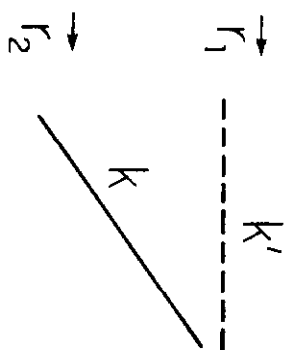
#### Figure Captions:

1. A space-time diagram for ultra-relativistic nuclear scattering. The two nuclei are incident along  $t = z$  and  $t = -z$ , and collide at  $t = z = 0$ . At some formation proper time  $\tau_f$  the particles thermalize and expand hydrodynamically until the decoupling time  $\tau_d$ .
2. The sum of the two paths which identical particles may take to generate a Hanbury-Brown-Twiss correlation.
3. The H-B-T correlation function for bosons and fermions. Notice that the correlation goes to zero for  $\Delta k \geq 1/R$  where  $R$  is the source size. The intercept at zero  $\Delta k$  is 2 for bosons and 0 for fermions.
4. The effect of a Coulomb correction to the H-B-T correlation function.
5. a) Pion production rate as a function of time in a cascade simulation. b) The spatial origin of the pions in the cascade simulation. c) The correlation between momentum and rapidity for the pion source.

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